

The Method of Variable Splitting

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In this talk, I shall explain the method of variable splitting and present some related proof complexity results. Very briefly, variable splitting is a method applicable to free-variable tableaux, free-variable sequent calculi, connection methods, and matrix characterizations, that reduces redundancies in the search space by exploiting a relationship between branching formulas and universal formulas. Using contextual information to differentiate between occurrences of free variables, the method admits conditions under which these occurrences may safely be assigned different values by substitutions or assignments. The method satisfies the following properties.

- Logically independent variable occurrences are allowed to be treated independently.
- Precise conditions under which local solutions may be combined into global solutions are stated.
- Search space redundancies caused by nonpermutabilities in standard free-variable calculi are removed.
- Dependencies between branching formulas and universal formulas are explicitly represented, analogous to Skolemization.
- A basis for branchwise search strategies and termination conditions in free-variable calculi is provided.
- Universal-variable methods are generalized.
- Novel characterizations of logical validity for first-order logic are defined.

Technically, this is achieved by labelling variable occurrences with labels identifying the context in which the variables occur. These labels are in turn used for determining the dependencies between formulas.

Besides motivating and explaining the method, I shall focus on two aspects: *soundness arguments* and *proof complexity*.

There are two typical ways of establishing soundness for tableau or sequent calculi: either by showing that the inferences of a derivation preserve a countermodel property or by transforming a proof in one calculus into a proof in another calculus known to be sound. Although not entirely straightforward, both of these may be used for proving soundness of variable splitting, and I shall give a brief overview of how this may be done.

The method of variable splitting gives an exponential speedup in terms of proof length in comparison with standard free-variable calculi. There are also further liberalizations of the method that give even stronger speedup results. In the talk, I shall give an outline of a few of these results.

References

- [Ant08] Roger Antonsen. *The Method of Variable Splitting*. PhD thesis, University of Oslo, Department of Informatics, June 2008.
- [AW07] Roger Antonsen and Arild Waaler. Liberalized Variable Splitting. *Journal of Automated Reasoning*, 38(1-3):3–30, 2007.